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Liquid Level Tracking Control of Three-tank Systems

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Abstract: In this paper, a liquid level tracking controller which is composed of a feedforward controller and a feedback controller is proposed for three-tank systems. Firstly, the flat property of three-tank systems is verified and a feedforward controller is designed accordingly so as to track the ideal trajectories. Secondly, in order to eliminate the tracking errors introduced by model uncertainties or unknown disturbances, a nonlinear model predictive controller is designed in which a terminal equality constraint is added for ensuring asymptotic convergence. Finally, the control performance is confirmed by both simulation and experiment results.

Key Words: Liquid Level Tracking, Three-tank Systems, Flat System, Nonlinear Model Predictive Controller

1 Introduction

The liquid level is one of the important controlled objects in modern process control [1] and the control accuracy plays an important role in improving product quality and enhancing economic benefits. Three-tank systems are typical multi-input multi-output (MIMO) systems with the features of strong coupling and nonlinearity, which make it of great research value in the study of liquid level control [2, 3].

Many control methods have been proposed for the liquid level tracking control problem. A neural network based PID controller is proposed in [4], and its dynamic performance is compared with a standard digital PID controller. It shows that the standard digital PID controller has fast response and a large overshoot. In contrast, neural network based PID controller can achieve better performance with the price of a relatively slow response. The control problem of three-tank systems is described as the disturbance attenuation problem of constrained linear systems [5]. The experiment results show that the designed closed-loop system can guarantee the attenuation performance of the system under the condition of satisfying time-domain constraints. However, the control accuracy of the proposed scheme can be guaranteed only when the expected liquid level is fixed.

Model predictive control (MPC) is able to deal with constraints of MIMO systems [6, 7], and to attenuate disturbances since the optimization problem is solved online with the new measurement. Compared to PID controller, MPC can achieve faster response and no overshoot [8]. Nonlinear model predictive control (NMPC) havs many advantages to handle the control problem of three-tank systems [9, 10]. However, its application has been limited due to the heavily computational burden and the difficulty of stability guarantee. A novel model predictive control scheme based on bees algorithm is proposed in [11] to reduce the calculational burden. However, the computational burden is still too heavy to implement. A RBF-ARX model-based MPC strategy is proposed in [12] where the computational burden is reduced by locally linearizing the model at each working point. Due to the model error caused by the linearization, the control accuracy is reduced inevitably.

In order to enhance the tracking control accuracy and avoid the model error caused by the linearization, a controller with a feedback control based on the property of differential flatness and a model predictive control of the error system is proposed. Through the ideal flat outputs, the ideal state trajectory and control input trajectory can be obtained [13]. The control input trajectory served as a feedforward can achieve fast tracking. A nonlinear model predictive controller is used as the feedback control to eliminate tracking errors. This two degrees of freedom control structure makes full use of not only the flatness of the system itself, but also the advantages of the NMPC. It is worth noting that with the effect of the feedforward control, the model error caused by linearization is avoided and the computational burden in NMPC is reduced accordingly.

The rest of this paper is organized as follows: Section 2 introduces the model of three-tank systems and sets up the control problem. The controller is designed in Section 3. Both the simulation and experiment results are shown in Section 4. Some conclusions are drawn in Section 5.

2 Problem Setup

This section includes two parts: the first part introduces the nonlinear model of a three-tank system and the second part describes the control problem.

As shown in Fig. 1, the three-tank system mainly consists of three tanks (Tank 1,Tank 2 and Tank 3) and two pumps (Pump 1 and Pump 2). Pump 1 and Pump 2 absorb liquid from the reservoir and supply the liquid respectively to Tank 1 and Tank 2. Tank 3 can only get liquid from Tank 1 and Tank 2 through the connecting pipes between them. The liquid in Tank 2 can inflow to the reservoir through the rightmost pipe. Other related parameters are referred in Table 1.

According to the Mass Balance Principle, the three-tank system is described as follows :

$$\begin{cases} S\dot{h}_1 = Q_1 - Q_{13} \\ S\dot{h}_2 = Q_2 + Q_{32} - Q_{20} \\ S\dot{h}_3 = Q_{13} - Q_{32}. \end{cases} \tag{1}$$

In terms of Torricelli Rule, Q_{13}, Q_{32} and Q_{20} are de-

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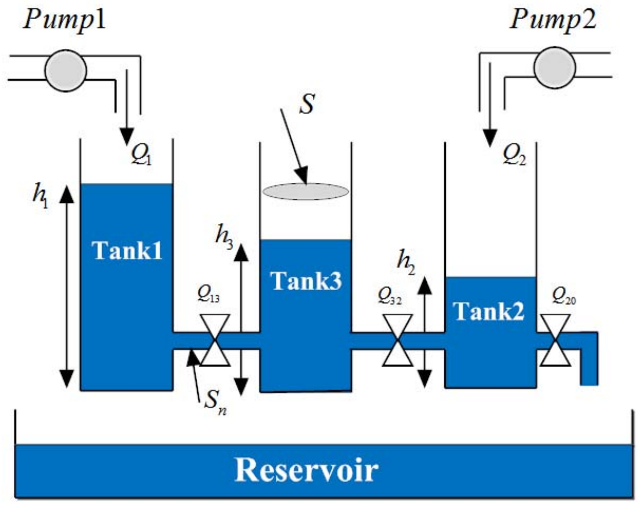

Fig. 1: Diagram of the three-tank system

Table 1: Symbols of the three-tank system

Symbol	Meaning
h_i	liquid level of Tank i $(i = 1, 2, 3)$
Q_j	flow rate from Pump j to Tank j $(j = 1, 2)$
Q_{13}	flow rate from Tank 1 to Tank 3
Q_{32}	flow rate from Tank 3 to Tank 2
Q_{20}	flow rate from Tank 2 to Reservoir
S	cross sectional area of Tank 1,2,3
S_n	cross sectional area of the connecting pipe

scribed by the following equations [3]:

$$\left\{ \begin{array}{l} Q_{13} = a_{z1} S_n \mathrm{sgn}\left(h_1 - h_3\right) \left(2g \left|h_1 - h_3\right|\right)^{1/2} \\ Q_{32} = a_{z3} S_n \mathrm{sgn}\left(h_3 - h_2\right) \left(2g \left|h_3 - h_2\right|\right)^{1/2} \\ Q_{20} = a_{z2} S_n (2gh_2)^{1/2} \end{array} \right. \tag{2}$$

where a_{zi} $(i = 1, 2, 3)$ represents the flow coefficients and g represents the gravitational acceleration.

Assuming that $h_1(t) \geq h_3(t) \geq h_2(t)$ for all $t \geq 0$, the three-tank system can be simplified:

$$\left\{ \begin{array}{l} S\dot{h}_1 = Q_1 - a_{z1} S_n (2g(h_1 - h_3))^{1/2} \\ S\dot{h}_2 = Q_2 + a_{z3} S_n (2g(h_3 - h_2))^{1/2} - a_{z2} S_n (2gh_2)^{1/2} \\ S\dot{h}_3 = a_{z1} S_n (2g(h_1 - h_3))^{1/2} - a_{z3} S_n (2g(h_3 - h_2))^{1/2}. \end{array} \right. \tag{3}$$

Choose $x = [h_1, h_2, h_3]^T$ as state of the three-tank system and $u = [Q_1, Q_2]^T$ as control input. The state and control input satisfy the following constraints:

$$0 \leq h_1, h_2, h_3 \leq H_{max} \tag{4}$$

$$0 \leq Q_1, Q_2 \leq Q_{max} \tag{5}$$

where H_{max} is the admissible liquid level of three tanks and Q_{max} is the maximum flow that the pumps can provide.

The control objective of the three-tank system is to track the ideal trajectories of h_1 and h_3 by regulation of control input u. At the same time, the state and control input constraints are taken into account.

3 Controller Design

In order to solve the control problem above, this section introduces a controller shown in the dashed box of Fig. 2. The controller includes a feedforward controller based on the theory of flat systems and a feedback controller (model predictive control). Denote h_1^* and h_3^* as the ideal trajectories of h_1 and h_3. Assume that h_1^* and h_3^* are finite order continuous. Denote x^f as the ideal trajectories of x obtained from the feedforward controller according to h_1^* and h_3^*, and x_e as the tracking errors coming from the model uncertainties or unknown disturbances. Denote u^f as the feedforward control which can make the system track the ideal trajectories x^f fast and u_e as the feedback control which aims at eliminating the tracking errors x_e.

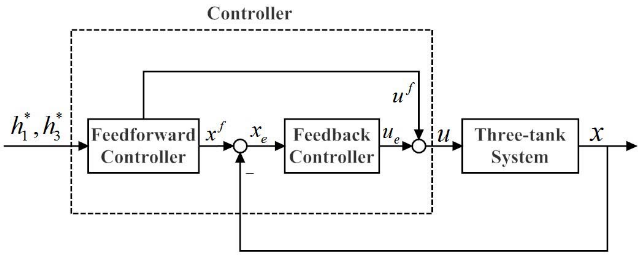

Fig. 2: Control block diagram of three-tank systems

3.1 Flat systems and feedforward control

Consider a system $\dot{\tilde{x}} = f(\tilde{x}, \tilde{u})$ with $\tilde{x} \in \Re^n$ and $\tilde{u} \in \Re^m$. If there exists $z \in \Re^m$ of the form $z = F\left(\tilde{x}, \tilde{u}, \dot{\tilde{u}}, ..., \tilde{u}^{(l)}\right)$ such that $\tilde{x} = \alpha(z, \dot{z}, ..., z^{(p)})$, $\tilde{u} = \varphi(z, \dot{z}, ..., z^{(q)})$, then the system is flat and z is called flat outputs [13] where functions f, F, α and φ are continuous and their finite-order derivatives exist, and letters l, p, q are positive integers. Therefore, the flat system is of the property that the state and input can be determined by flat outputs and their finite-order derivatives [13]. In general, the state variables of systems are chosen to test whether the selected state variables satisfy the definition of flat outputs so as to take advantage of the property of flat systems and design the feedforward controller [14].

Three tank systems are flat systems and the flat outputs are not unique [15]. Because of the ideal trajectories of h_1 and h_3 are known, h_1 and h_3 can be used as flat outputs, i.e., $z = [h_1, h_3]^T$. In the following, the flat property of the three-tank system will be tested and x^f and u^f will be obtained.

According to the system (3), h_2 can be expressed as:

$$h_2 = h_3 - \frac{1}{2g} \left(\frac{a_{z1} S_n \sqrt{2g(h_1 - h_3)} - S\dot{h}_3}{a_{z3} S_n} \right)^2. \tag{6}$$

Furthermore, Q_1 and Q_2 can be expressed as:

$$\left\{ \begin{array}{l} Q_1 = S\dot{h}_1 + a_{z1} S_n \sqrt{2g(h_1 - h_3)} \\ Q_2 = S\dot{h}_2 - a_{z3} S_n \sqrt{2g(h_3 - h_2)} + a_{z2} S_n \sqrt{2gh_2}. \end{array} \right. \tag{7}$$

It is not difficult to find that h_1 and h_3 satisfy the definition of flat outputs and the flat property of the three-tank system is verified. So, the ideal flat outputs can be used to design x^f and u^f. At time instant t, denote the ideal flat outputs as $z^* = [h_1^*(t), h_3^*(t)]^T$, and $x^f(t) =$

$\left[x_1^f(t), x_2^f(t), x_3^f(t)\right]^T$, where

$$\begin{cases} x_1^f(t) = h_1^*(t) \\ x_3^f(t) = h_3^*(t) \\ x_2^f(t) = h_3^*(t) - \dfrac{1}{2g(a_{z3}S_n)^2} \\ \qquad * \left(a_{z1}S_n\sqrt{2g\left(h_1^*(t) - h_3^*(t)\right)} - S\dot{h}_3^*(t)\right)^2 \end{cases} \tag{8}$$

and the feedforward control input $u^f(t) = \left[u_1^f(t), u_2^f(t)\right]^T$, where

$$\begin{cases} u_1^f(t) = S\dot{h}_1^*(t) + a_{z1}S_n\sqrt{2g\left(h_1^*(t) - h_3^*(t)\right)} \\ u_2^f(t) = S\dot{x}_2^f(t) - a_{z3}S_n\sqrt{2g\left(h_3^*(t) - x_2^f(t)\right)} \\ \qquad + a_{z2}S_n\sqrt{2gx_2^f(t)}. \end{cases} \tag{9}$$

3.2 Nonlinear model predictive controller

Due to model uncertainties or other unknown disturbances, tracking errors can not be avoided. Define $x_e = [x_{e1}, x_{e2}, x_{e3}]^T$ as state of the error system and $u_e = [u_{e1}, u_{e2}]^T$ as control input, where

$$\begin{cases} x_{e1} = x_1^f - h_1 \\ x_{e2} = x_2^f - h_2 \\ x_{e3} = x_3^f - h_3 \\ u_{e1} = Q_1 - u_1^f \\ u_{e2} = Q_2 - u_2^f. \end{cases} \tag{10}$$

The error system can be described as follows:

$$\begin{cases} \dot{x}_{e1} = \dot{x}_1^f - \dot{h}_1 \\ \dot{x}_{e2} = \dot{x}_2^f - \dot{h}_2 \\ \dot{x}_{e3} = \dot{x}_3^f - \dot{h}_3 \end{cases} \tag{11}$$

where $\dot{h}_1, \dot{h}_2$ and $\dot{h}_3$ can be expressed according to Eq.(3) and Eq.(10):

$$\begin{cases} \dot{h}_1 = \frac{u_{e1}+u_1^f}{S} - \frac{a_{z1}S_n}{S}\left(2g\left(x_{e1} - x_{e3} + x_1^f - x_3^f\right)\right)^{1/2} \\ \dot{h}_2 = \frac{u_{e2}+u_2^f}{S} + \frac{a_{z3}S_n}{S}\left(2g\left(x_{e3} - x_{e2} + x_3^f - x_2^f\right)\right)^{1/2} \\ \qquad - \frac{a_{z2}S_n}{S}\left(2g\left(x_{e2} + x_2^f\right)\right)^{1/2} \\ \dot{h}_3 = \frac{a_{z1}S_n}{S}\left(2g\left(x_{e1} - x_{e3} + x_1^f - x_3^f\right)\right)^{1/2} - \frac{a_{z3}S_n}{S} \\ \qquad *\left(2g\left(x_{e3} - x_{e2} + x_3^f - x_2^f\right)\right)^{1/2} \end{cases} \tag{12}$$

and $\dot{x}_1^f, \dot{x}_2^f$ and $\dot{x}_3^f$ can be expressed according to Eq.(8):

$$\begin{cases} \dot{x}_1^f = \dot{h}_1^* \\ \dot{x}_3^f = \dot{h}_3^* \\ \dot{x}_2^f = \dot{h}_3^* - \left(\dfrac{a_{z1}S_n\sqrt{2g\left(h_1^* - h_3^*\right)} - S\dot{h}_3^*}{g(a_{z3}S_n)^2}\right) \\ \qquad * \left(a_{z1}S_n\sqrt{2g}\dfrac{\dot{h}_1^* - \dot{h}_3^*}{2\sqrt{h_1^* - h_3^*}} - S\ddot{h}_3^*\right). \end{cases} \tag{13}$$

Combining Eq.(12) and Eq.(13), the error system (11) is written in the following form:

$$\dot{x}_e = f_e(x_e, u_e) \tag{14}$$

where the function f_e is parameter-dependent on the ideal flat outputs and their finite-order derivatives.

The function $f_e(x_e, u_e) = 0$ while $x_e = [0,0,0]^T$, $u_e = [0,0]^T$. That is to say, $[0,0,0]^T$ is the equilibrium of the error system (14).

In order to make the tracking errors converge to zero in the framework of model predictive control [16, 17], the following online optimization problem is solved at each time instant t :

Problem 1

$$\min_{U_t} J(x_e(t), U_t) \tag{15}$$

st.

$$\begin{aligned} &\dot{\bar{x}}_e(\tau) = f_e(\bar{x}_e(\tau), \bar{u}_e(\tau)), t \le \tau \le t + T_p \\ &0 \le \bar{u}_e(\tau) + u^f(\tau) \le Q_{max}, t \le \tau \le t + T_p \\ &0 \le x^f(\tau) - \bar{x}_e(\tau) \le H_{max}, t < \tau \le t + T_p \\ &\bar{x}_e(t) = x_e(t) \\ &\bar{x}_e(t + T_p) = 0 \end{aligned}$$

with

$J(x_e(t), U_t) = \int_t^{t+T_p}\left(||\bar{x}_e(\tau)||_Q^2 + ||\bar{u}_e(\tau)||_R^2\right)d\tau$.

In Problem 1, $x_e(t)$ is the error state at time t, $U_t := u_e(\cdot, x_e(t))$ denotes the control input trajectory related to $x_e(t)$. $\bar{u}_e(\tau)$ is the predicted control input for all $\tau \in [t, t+T_p]$ and $\bar{u}_e(\tau) = u_e(\tau, x_e(t))$. Both Q and R are positive definite weighting matrices with appropriate dimensions, T_p is the prediction horizon. Problem 1 is solved in discrete time with a sampling of δ.

Denote U_t^* as the optimal solution of the Problem 1, that is $U_t^* = u_e^*(\cdot, x_e(t))$, then the control input of the NMPC at time instant t can be denoted as $u_e(t)$,

$$u_e(t) = u_e^*(t, x_e(t)). \tag{16}$$

So, the final control input at time instant t is $u(t)$, and

$$u(t) = u^f(t) + u_e(t). \tag{17}$$

The proposed control law can be formally obtained by Algorithm 1.

Algorithm 1 Tracking control algorithm for three-tank systems

1: **while** $(t \le 2500)$ **do**
2: Calculate $x^f(t)$ and $u^f(t)$ via (8)-(9) at time instant t ;
3: Measure system state $x(t)$ at time instant t;
4: Calculate the state $x_e(t)$ via (10);
5: Solve Problem 1 to get $u_e(t)$ via (16);
6: Take the value $u(t)$ calculated via (17) as the current control input of the system until the next sampling time $t + \delta$;
7: At time $t + \delta$, set $t := t + \delta$;
8: **end while**

4 Simulation and Experiment

In this section, simulation and experiment are carried out separately for the three-tank system over 2500 seconds. The ideal trajectories are $h_1^*(t)$ and $h_3^*(t)$, where

$$h_1^*(t) = \begin{cases} 50\sin(\pi t/1000) + 1.5, t \in [0, 500] \\ 4\sin(\pi t/1000) + 47.5, t \in (500, 2500], \end{cases}$$

and

$$h_3^*(t) = \begin{cases} 35\sin(\pi t/1000) + 0.6, t \in [0, 500] \\ 4\sin(\pi t/1000) + 31.6, t \in (500, 2500]. \end{cases}$$

The sampling time $\delta = 1s$. Other related parameters of the system and controller are listed in Table 2. The simulation results are shown in Fig. 3-5 and the experiment results are shown in Fig. 6-8, respectively.

Suppose that there is no measurement noise or model mismatch in the simulation. Both states and control inputs constraints are satisfied which are shown in Fig. 3 and Fig. 5. The evolution of tracking errors are shown in Fig. 4. At the initial time instant, both tracking errors and control inputs are large due to the large initial differences between the real states and the ideal states. After about 500 seconds, the tracking errors are kept within 0.3 cm. Accordingly, the control inputs vary smoothly. So, the proposed scheme can track the ideal trajectories with high accuracy.

Table 2: Parameters and Values

Parameter	Value
R	[1,0;0,1]
Q	[100,0,0;0,200,0;0,0,1000]
T_p	$1s$
Q_{max}	$145ml/s$
H_{max}	$60cm$
S	$154cm^2$
S_n	$0.5\ cm^2$
g	$981ml/s^2$
a_{z1}	0.3465
a_{z2}	0.7301
a_{z3}	0.5130

Compared with simulation results, an experiment with the same parameters as simulation achieves the similar results, c.f. Fig. 6-8. But, the tracking errors are larger than the results in simulation due to the existence of model errors and large measurement noises. In addition, the frequent fluctuation of measurement noises directly lead to the fluctuation of the pumps which is shown in Fig. 8. The maximum measurement noise is about 2 cm which can be seen from Fig. 6, and the tracking errors are kept within 2 cm shown in Fig. 7. Although the measurement noises are large to some extent, the proposed scheme can still track the ideal trajectories in a satisfied way.

5 Conclusion

A practical liquid level tracking control method was proposed in this paper. Feedforward controller was obtained by the flat outputs utilizing the flatness property of three tank systems. Model predictive control, a feedback control method, keeps the system dynamics in the small area around the trajectory. Compared with the schemes which linearized the system around the trajectory, the proposed scheme can achieve high accuracy, and deal with state and input constraints directly.

Future research will focus on reducing the effects of measurement noises or model mismatches which lead to large errors in the experiment.

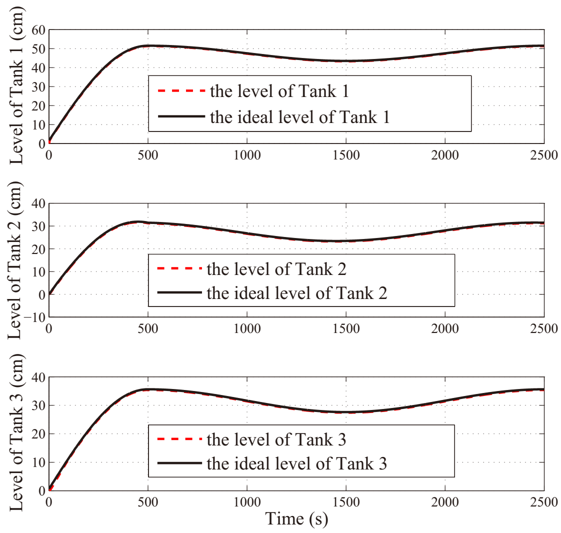

Fig. 3: The evolution of states in simulation

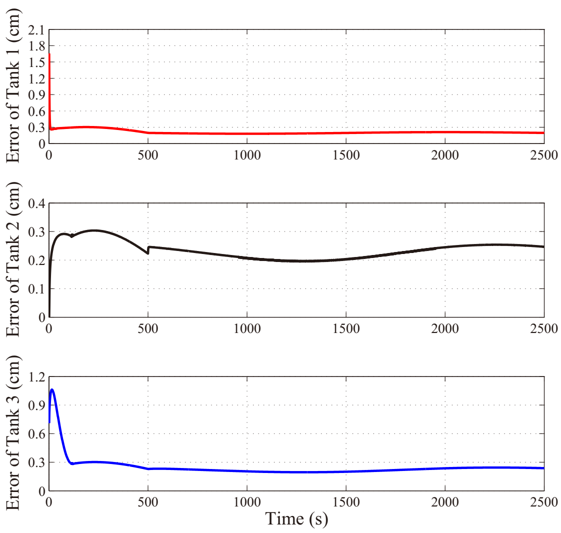

Fig. 4: The evolution of tracking errors in simulation

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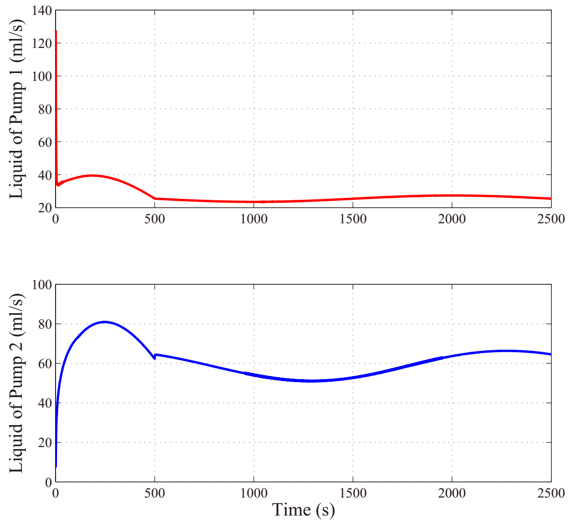

Fig. 5: The evolution of inputs in simulation

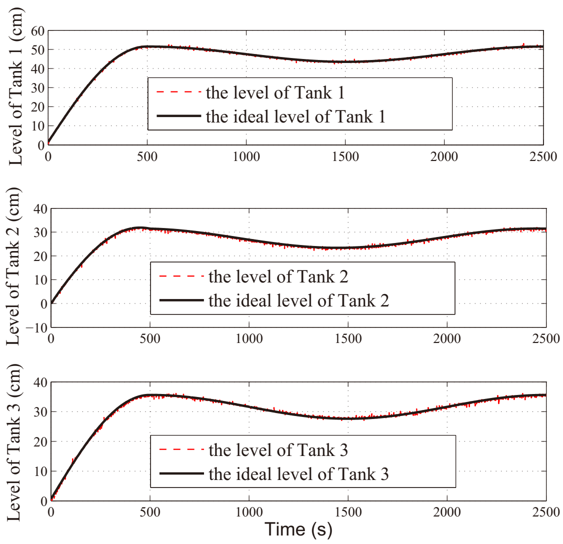

Fig. 6: The evolution of states in experiment

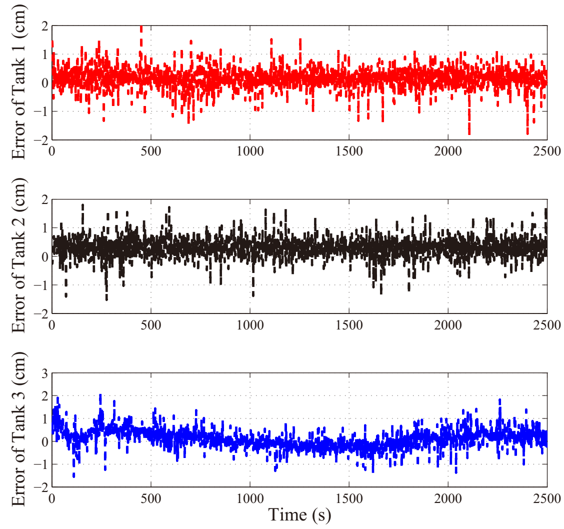

Fig. 7: The evolution of tracking errors in experiment

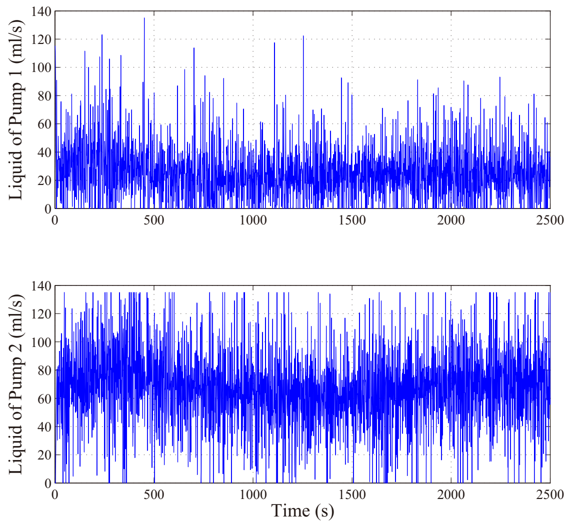

Fig. 8: The evolution of inputs in experiment

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